Introduction

- This chapter deals with 3 equations commonly used in fluid mechanics
  - *The mass equation* is an expression of the conservation of mass principle.
  - *The Bernoulli equation* is concerned with the conservation of kinetic, potential, and flow energies of a fluid stream and their conversion to each other.
  - *The energy equation* is a statement of the conservation of energy principle.

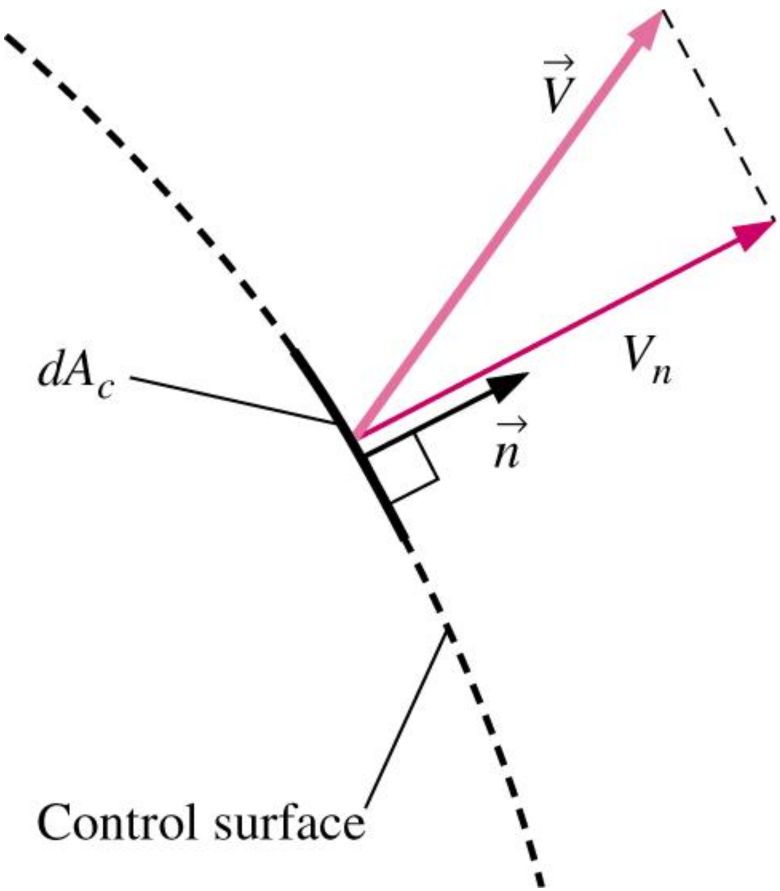
Objectives

- After completing this chapter, you should be able to
  - Apply the mass equation to balance the incoming and outgoing flow rates in a flow system.
  - Recognize various forms of mechanical energy, and work with energy conversion efficiencies.
  - Understand the use and limitations of the Bernoulli equation, and apply it to solve a variety of fluid flow problems.
  - Work with the energy equation expressed in terms of heads, and use it to determine turbine power output and pumping power requirements.

Conservation of Mass

- Conservation of mass principle is one of the most fundamental principles in nature.
- Mass, like energy, is a conserved property, and it cannot be created or destroyed during a process.
- For *closed systems* mass conservation is implicit since the mass of the system remains constant during a process.
- For *control volumes*, mass can cross the boundaries which means that we must keep track of the amount of mass entering and leaving the control volume.

Mass and Volume Flow Rates

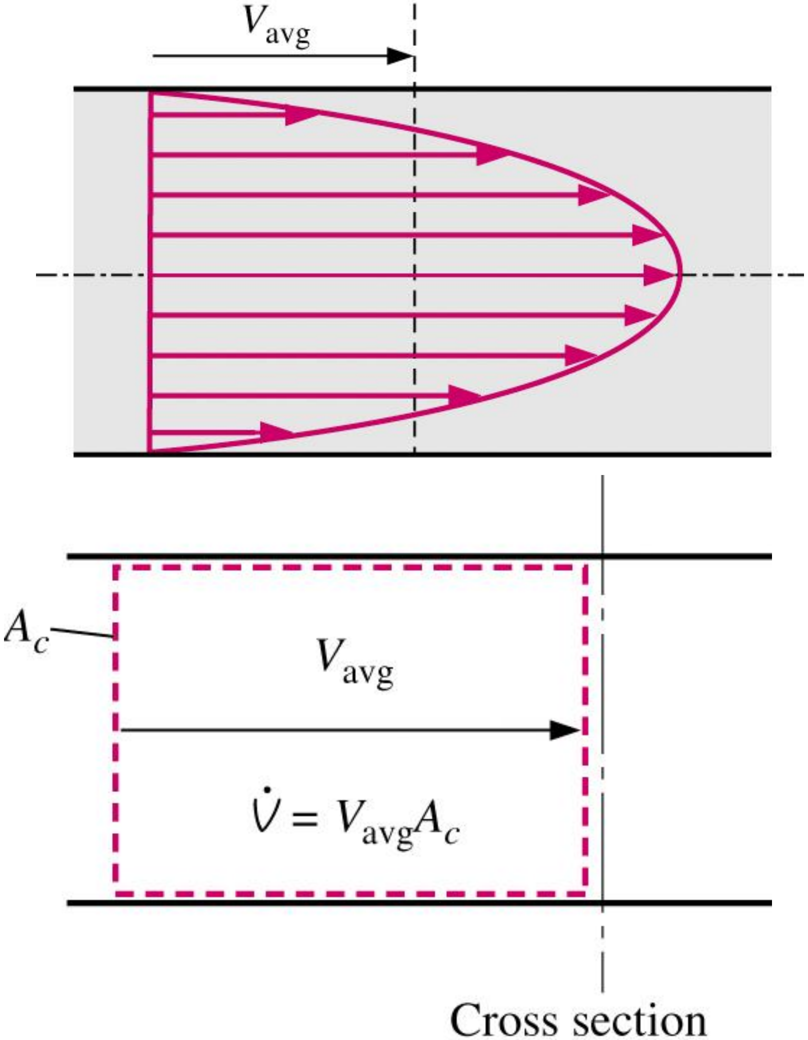


- The amount of mass flowing through a control surface per unit time is called the **mass flow rate** and is denoted $\dot{m}$
- The dot over a symbol is used to indicate *time rate of change*.
- Flow rate across the entire cross-sectional area of a pipe or duct is obtained by integration

$$\dot{m} = \int_{A_c} \delta m = \int_{A_c} \rho V_n dA_c$$

- While this expression for $\dot{m}$ is exact, it is not always convenient for engineering analyses.

Average Velocity and Volume Flow Rate



- Integral in $\dot{m}$ can be replaced with average values of ρ and V_n

$$V_{avg} = \frac{1}{A_c} \int_{A_c} V_n \, dA_c$$

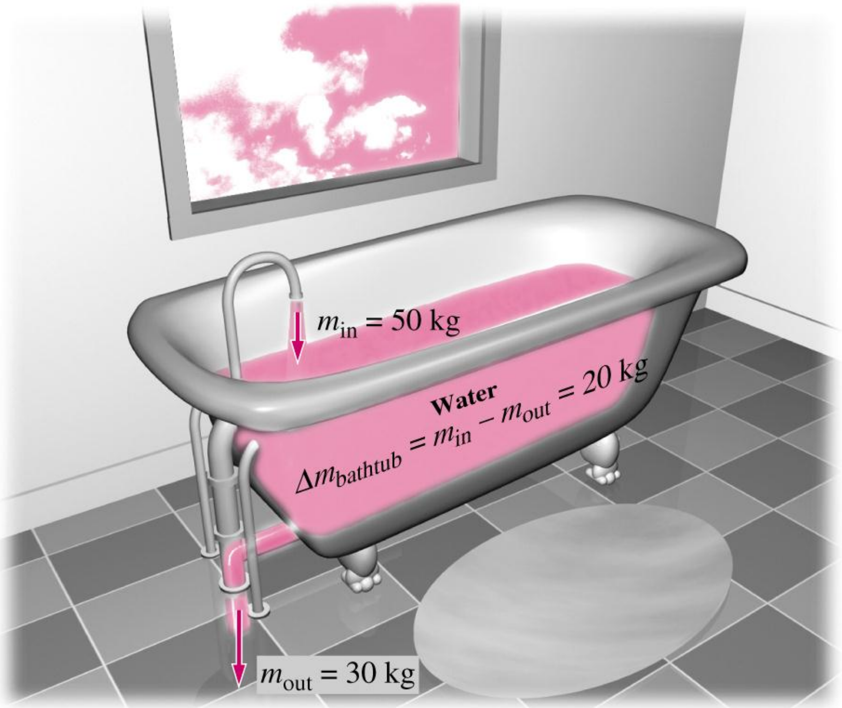
- For many flows variation of ρ is very small: $\dot{m} = \rho V_{avg} A_c$
- Volume flow rate $\dot{V}$ is given by

$$\dot{V} = \int_{A_c} V_n \, dA_c = V_{avg} A_c = V A_c$$

- Note: many textbooks use Q instead of $\dot{V}$ for volume flow rate.
- Mass and volume flow rates are related by

$$\dot{m} = \rho \dot{V}$$

Conservation of Mass Principle

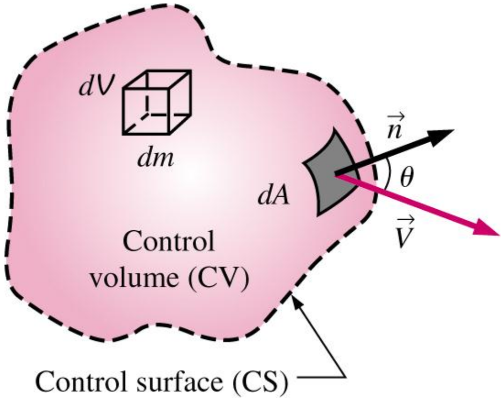


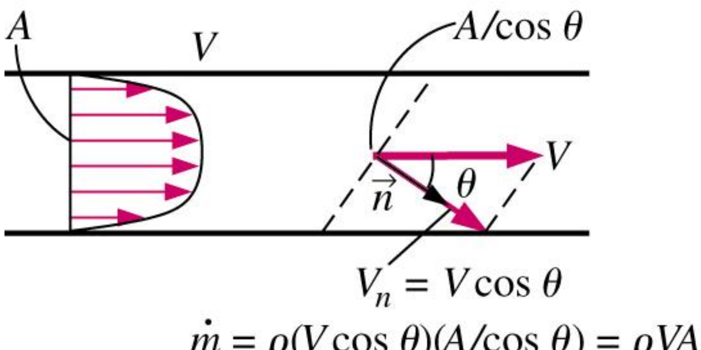
- The **conservation of mass principle** can be expressed as

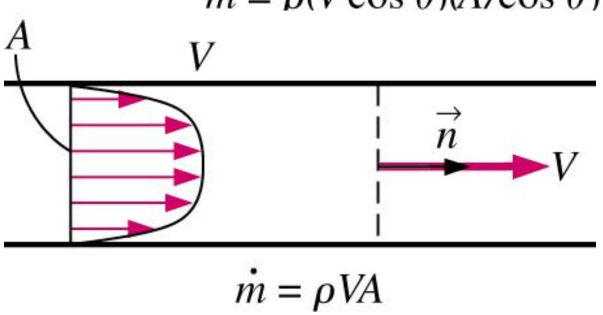
$$\dot{m}_{in} - \dot{m}_{out} = \frac{dm_{CV}}{dt}$$

- Where $\dot{m}_{in}$ and $\dot{m}_{out}$ are the total rates of mass flow into and out of the CV, and dm_{CV}/dt is the rate of change of mass within the CV.

Conservation of Mass Principle







- For CV of arbitrary shape,
  - rate of change of mass within the CV

$$\frac{dm_{CV}}{dt} = \frac{d}{dt}\int_{CV} \rho \, d\forall$$

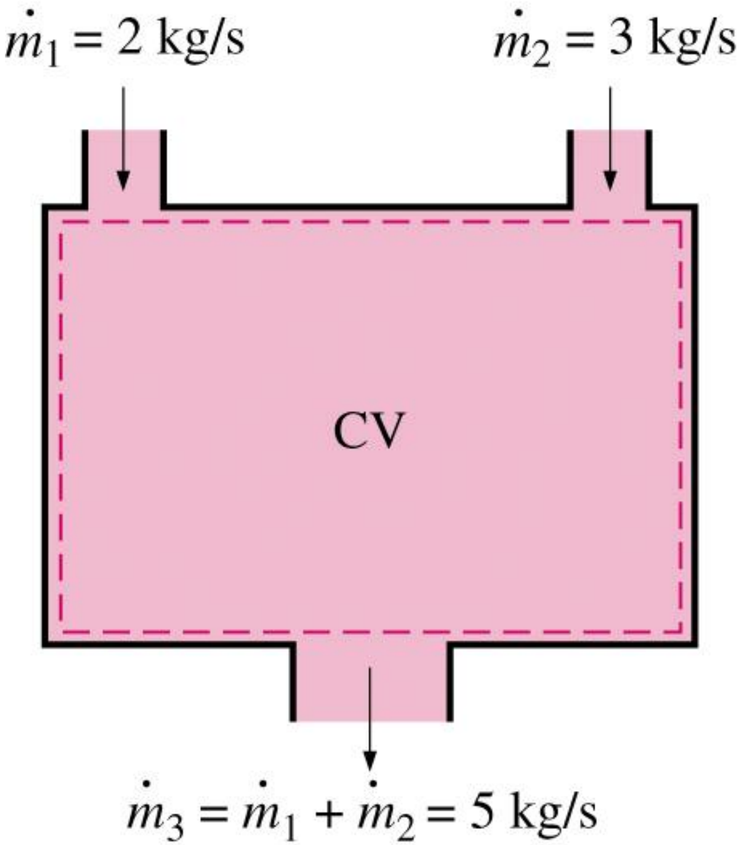
  - net mass flow rate

$$\dot{m}_{net} = \int_{CS} \delta\dot{m} = \int_{CS} \rho V_n dA = \int_{CS} \rho\left(\vec{V} \cdot \vec{n}\right) dA$$

- Therefore, general conservation of mass for a fixed CV is:

$$\frac{d}{dt}\int_{CV} \rho \, d\forall + \int_{CS} \rho\left(\vec{V} \cdot \vec{n}\right) dA = 0$$

Steady—Flow Processes



- For steady flow, the total amount of mass contained in CV is constant.
- Total amount of mass entering must be equal to total amount of mass leaving

$$\sum_{in} \dot{m} = \sum_{out} \dot{m}$$

- For incompressible flows,

$$\sum_{in} V_n A_n = \sum_{out} V_n A_n$$

Mechanical Energy

- **Mechanical energy** can be defined as *the form of energy that can be converted to mechanical work completely and directly by an ideal mechanical device such as an ideal turbine.*
- Flow P/ρ, kinetic V^2/g, and potential gz energy are the forms of mechanical energy $e_{mech} = P/\rho + V^2/g + gz$
- Mechanical energy change of a fluid during incompressible flow becomes

$$\Delta e_{mech} = \frac{P_2 - P_1}{\rho} + \frac{V_2^{\,2} - V_1^{\,2}}{2} + g\left(z_2 - z_1\right)$$

- In the absence of loses, Δe_{mech} represents the work supplied to the fluid ($\Delta e_{mech}>0$) or extracted from the fluid ($\Delta e_{mech}<0$).

Efficiency

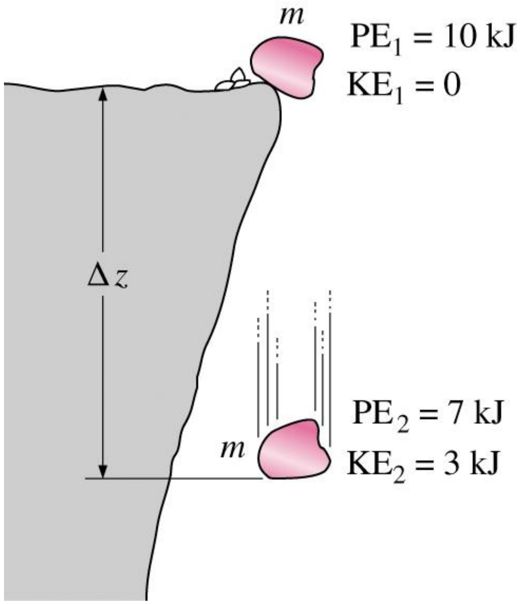
- Transfer of e_{mech} is usually accomplished by a rotating shaft: *shaft work*
- Pump, fan, propulsor: receives shaft work (e.g., from an electric motor) and transfers it to the fluid as mechanical energy
- Turbine: converts e_{mech} of a fluid to shaft work.
- In the absence of irreversibilities (e.g., friction), **mechanical efficiency** of a device or process can be defined as

$$\eta_{mech} = \frac{E_{mech,out}}{E_{mech,in}} = 1 - \frac{E_{mech,loss}}{E_{mech,in}}$$

- If η_{mech} < 100%, losses have occurred during conversion.

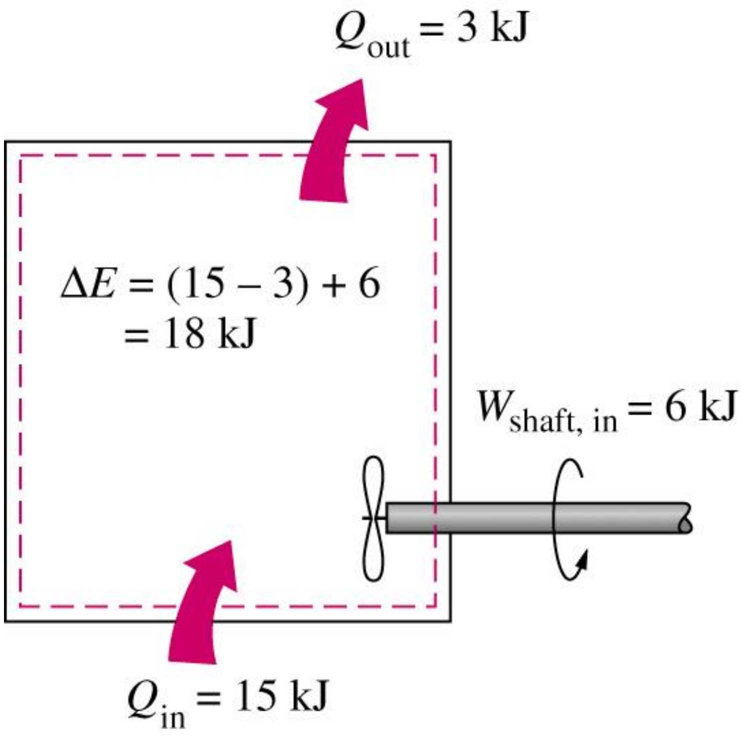
General Energy Equation

- One of the most fundamental laws in nature is the **1st law of thermodynamics**, which is also known as the **conservation of energy principle**.
- It state that *energy can be neither created nor destroyed during a process; it can only change forms*



- Falling rock, picks up speed as PE is converted to KE.
- If air resistance is neglected, PE + KE = constant

General Energy Equation



- The energy content of a closed system can be changed by two mechanisms: *heat transfer Q* and *work transfer W*.
- Conservation of energy for a closed system can be expressed in rate form as

$$\dot{Q}_{net,in} + \dot{W}_{net,in} = \frac{dE_{sys}}{dt}$$

- Net rate of heat transfer to the system:

$$\dot{Q}_{net,in} = \dot{Q}_{in} - \dot{Q}_{out}$$

- Net power input to the system:

$$\dot{W}_{net,in} = \dot{W}_{in} - \dot{W}_{out}$$

General Energy Equation

- Recall general RTT

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho b \, d\forall + \int_{CS} \rho b \left(\vec{V}_r \cdot \vec{n}\right) dA$$

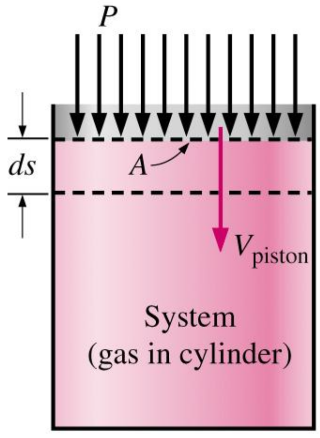
- "Derive" energy equation using *B*=*E* and *b*=*e*

$$\frac{dE_{sys}}{dt} = \dot{Q}_{net,in} + \dot{W}_{net,in} = \frac{d}{dt}\int_{CV} \rho e \, d\forall + \int_{CS} \rho e \left(\vec{V}_r \cdot \vec{n}\right) dA$$

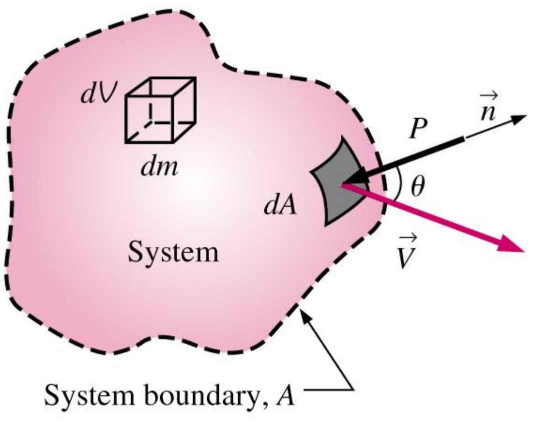
- Break power into rate of shaft and pressure work

$$\dot{W}_{net,in} = \dot{W}_{shaft,net,in} + \dot{W}_{pressure,net,in} = \dot{W}_{shaft,net,in} - \int P \left(\vec{V} \cdot \vec{n}\right) dA$$

General Energy Equation



(a)



(b)

- Where does expression for pressure work come from?
- When piston moves down *ds* under the influence of *F=PA*, the work done on the system is $\delta W_{boundary} = PAds$.
- If we divide both sides by *dt*, we have

$$\delta \dot{W}_{pressure} = \delta \dot{W}_{boundary} = PA\frac{ds}{dt} = PAV_{piston}$$

- For generalized control volumes:

$$\delta \dot{W}_{pressure} = -PdAV_n = -PdA\left(\vec{V} \cdot \vec{n}\right)$$

- Note sign conventions:
  - $\vec{n}$ is outward pointing normal
  - Negative sign ensures that work done is positive when is done *on* the system.

General Energy Equation

- Moving integral for rate of pressure work to RHS of energy equation results in:

$$\dot{Q}_{net,in} + \dot{W}_{shaft,net,in} = \frac{d}{dt}\int_{CV} \rho e \, d\forall + \int_{CS} \left(\frac{P}{\rho} + e \right) \rho \left(\vec{V}_r \cdot \vec{n} \right) dA$$

- Recall that P/ρ is the **flow work,** which is the work associated with pushing a fluid into or out of a CV per unit mass.

General Energy Equation

- As with the mass equation, practical analysis is often facilitated as averages across inlets and exits

$$\dot{Q}_{net,in} + \dot{W}_{shaft,net,in} = \frac{d}{dt}\int_{CV} \rho e \, d\forall + \sum_{out} \dot{m}\left(\frac{P}{\rho} + e\right) - \sum_{in} \dot{m}\left(\frac{P}{\rho} + e\right)$$

$$\dot{m} = \int_{A_C} \rho \left(\vec{V} \cdot \vec{n}\right) dA_c$$

- Since $e=u+ke+pe = u+V^2/2+gz$

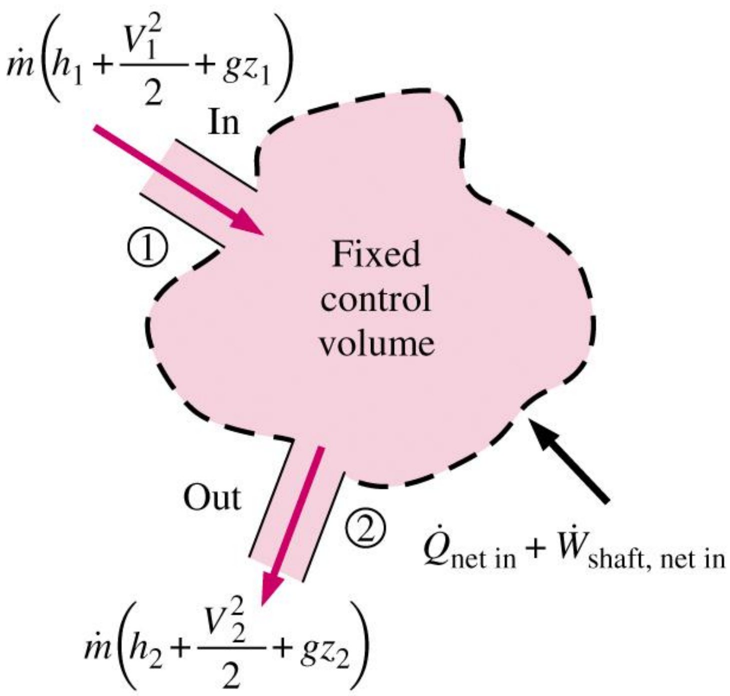
$$\dot{Q}_{net,in} + \dot{W}_{shaft,net,in} = \frac{d}{dt}\int_{CV} \rho e \, d\forall + \sum_{out} \dot{m}\left(\frac{P}{\rho} + u + \frac{V^2}{2} + gz\right) - \sum_{in} \dot{m}\left(\frac{P}{\rho} + u + \frac{V^2}{2} + gz\right)$$

Energy Analysis of Steady Flows

$$Q_{net,in} + W_{shaft,net,in} = \sum_{out} \dot{m}\left(h + \frac{V^2}{2} + gz \right) - \sum_{in} \dot{m}\left(h + \frac{V^2}{2} + gz \right)$$

- For steady flow, time rate of change of the energy content of the CV is zero.
- This equation states: *the net rate of energy transfer to a CV by heat and work transfers during steady flow is equal to the difference between the rates of outgoing* and incoming energy flows with mass.

Energy Analysis of Steady Flows



- For **single-stream devices**, mass flow rate is constant.

$$q_{net,in} + w_{shaft,net,in} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g\left(z_2 - z_1\right)$$

$$w_{shaft,net,in} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + \left(u_2 - u_1 - q_{net,in}\right)$$

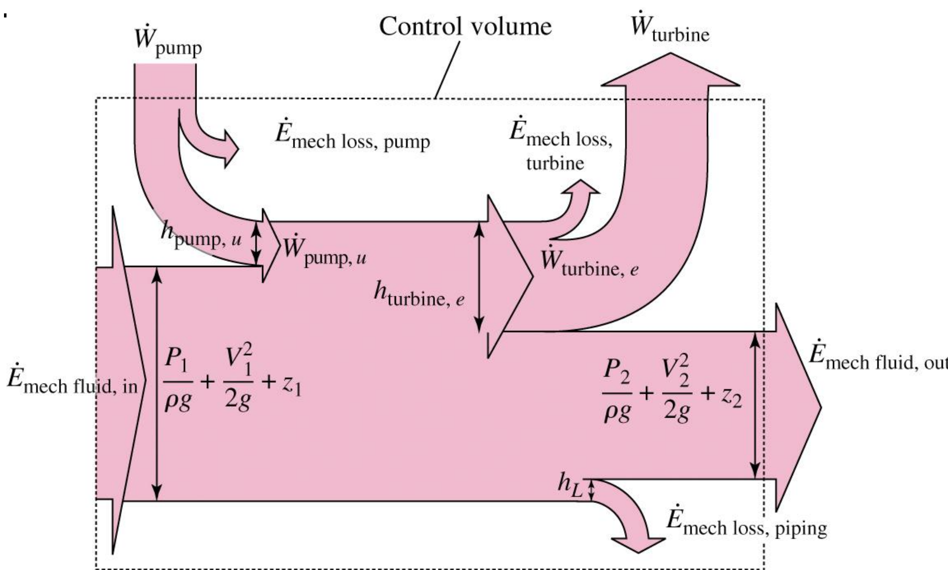
$$\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 + w_{pump} = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + w_{turbine} + e_{mech,loss}$$

Energy Analysis of Steady Flows

- Divide by *g* to get each term in units of length

$$\frac{P_1}{\rho_1 g} + \frac{V_1^{\,2}}{2g} + z_1 + h_{pump} = \frac{P_2}{\rho_2 g} + \frac{V_2^{\,2}}{2g} + z_2 + h_{turbine} + h_L$$

Magnitude of each term is now expressed as an equivalen



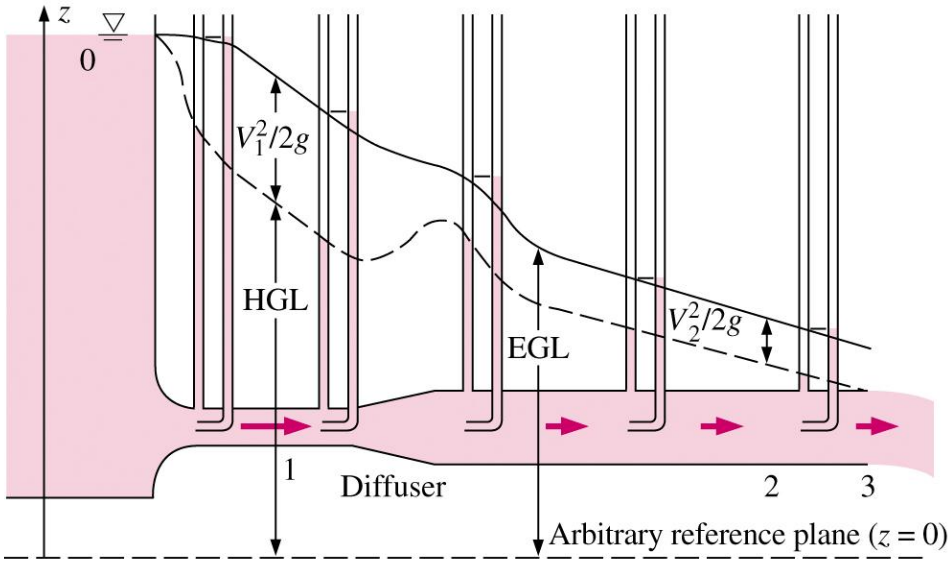
The Bernoulli Equation

- If we neglect piping losses, and have a system without pumps or turbines

$$\frac{P_1}{\rho_1 g} + \frac{V_1^{\,2}}{2g} + z_1 = \frac{P_2}{\rho_2 g} + \frac{V_2^{\,2}}{2g} + z_2$$

- This is the **Bernoulli equation**
- It can also be derived using Newton's second law of motion.
- 3 terms correspond to: Static, dynamic, and hydrostatic head (or pressure).

HGL and EGL



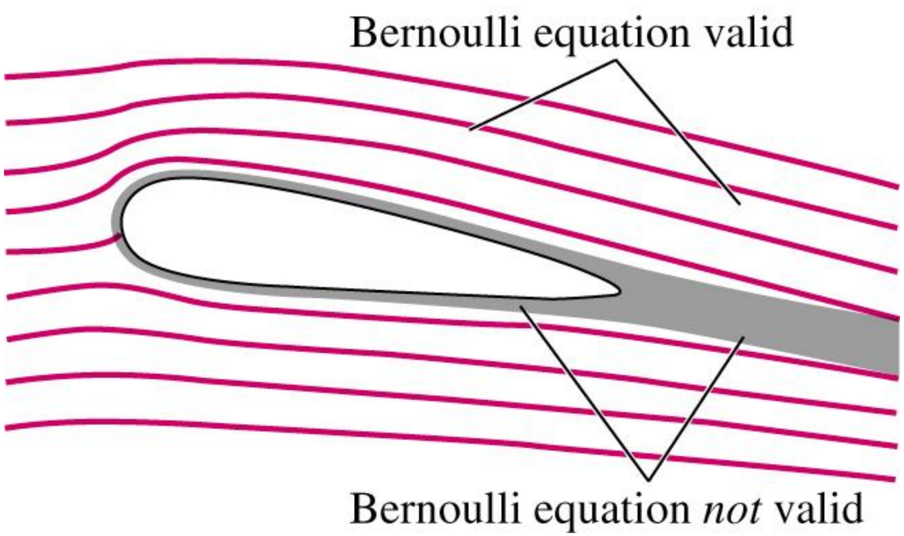
- It is often convenient to plot mechanical energy graphically using heights.
- Hydraulic Grade Line

$$HGL = \frac{P}{\rho g} + z$$

- Energy Grade Line (or total energy)

$$EGL = \frac{P}{\rho g} + \frac{V^2}{2g} + z$$

The Bernoulli Equation



- The **Bernoulli equation** is an *approximate relation between pressure, velocity*, and *elevation and is valid in regions of steady, incompressible flow where net frictional forces are negligible.*
- Equation is useful in flow regions outside of boundary layers and wakes.

The Bernoulli Equation

- Limitations on the use of the Bernoulli Equation
  - Steady flow: $d/dt = 0$
  - Frictionless flow
  - No shaft work: $w_{pump} = w_{turbine} = 0$
  - Incompressible flow: ρ = constant
  - No heat transfer: $q_{net,in} = 0$
  - Applied along a streamline (except for irrotational flow)