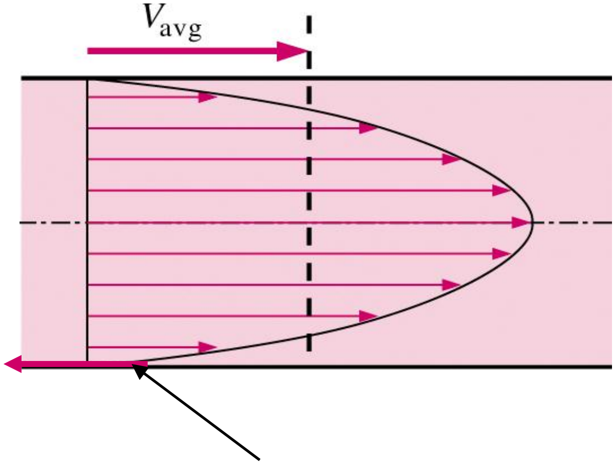
Additional Lecture No. 6 - Flow in Pipes

Objectives

1. Have a deeper understanding of laminar and turbulent flow in pipes and the analysis of fully developed flow
2. Calculate the major and minor losses associated with pipe flow in piping networks and determine the pumping power requirements
3. Understand the different velocity and flow rate measurement techniques and learn their advantages and disadvantages

Introduction



Friction force of wall on fluid

- Average velocity in a pipe
  - Recall - because of the <u>no-slip condition</u>, the velocity at the walls of a pipe or duct flow is zero
  - We are often interested only in V_{avg}, which we usually call just V (drop the subscript for convenience)
  - Keep in mind that the no-slip condition causes shear stress and <u>friction</u> along the pipe walls

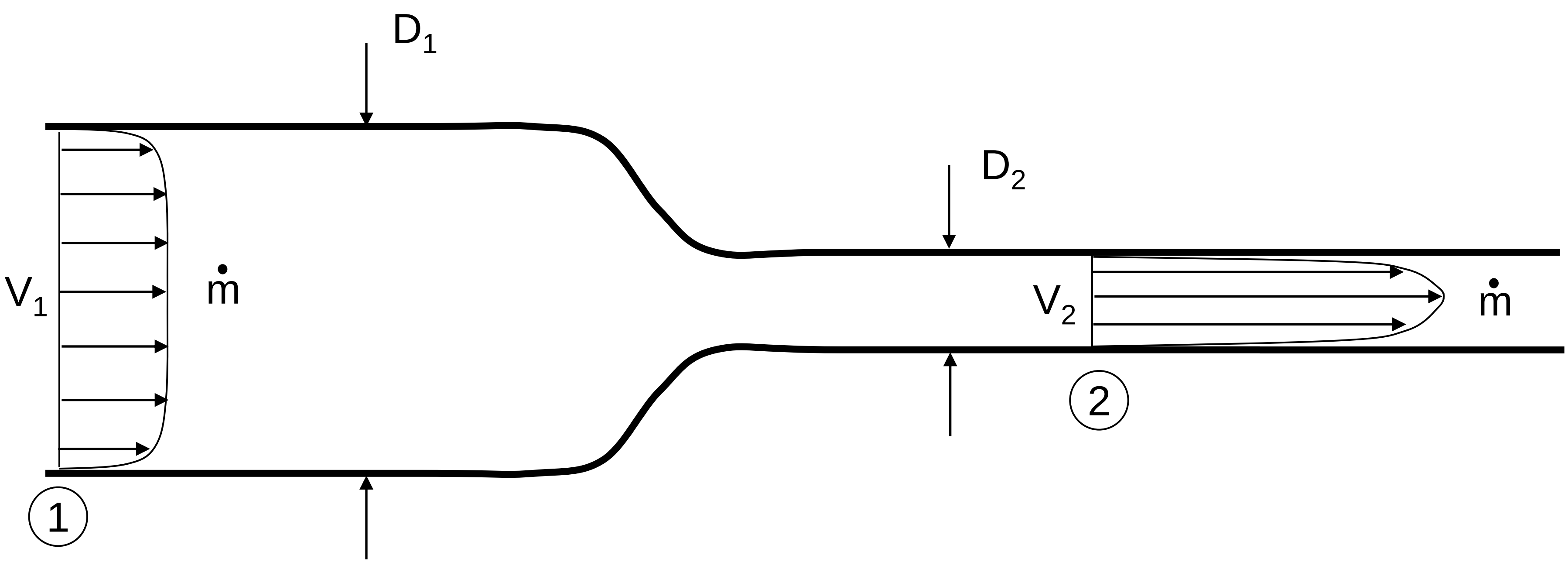
Introduction

- For pipes of constant diameter and incompressible flow
  - V_{avg} stays the same down the pipe, even if the velocity profile changes
    - Why? Conservation of

$$\dot{m} = \underset{\text{same}}{\rho}\,\underset{\text{same}}{V_{avg}}\,\underset{\text{same}}{A} = constant$$

Introduction

- For pipes with variable diameter, $\dot{m}$ is still the same due to conservation of mass, but $V_1 \neq V_2$



Laminar and Turbulent Flows

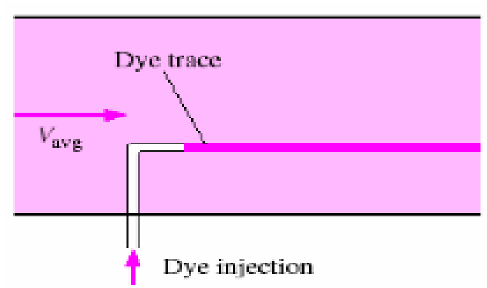
Laminar Flow

Can be steady or unsteady.

(Steady means that the flow field at any instant in time is the same as at any other instant in time.)

Can be one-, two-, or three-dimensional.

Has regular, *predictable* behavior



Analytical solutions are possible (see Chapter 9).

Occurs at *low* Reynolds numbers.

Turbulent Flow

Is always *unsteady*.

Why? There are always random, swirling motions (vortices or eddies) in a turbulent flow.

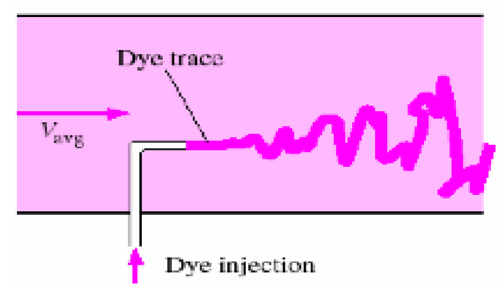
Note: However, a turbulent flow can be steady *in the mean*. We call this a *stationary turbulent flow*.

Is always *three-dimensional*.

Why? Again because of the random swirling eddies, which are in all directions.

Note: However, a turbulent flow can be 1-D or 2-D *in the mean*.

Has irregular or *chaotic* behavior (cannot predict exactly – there is some randomness associated with any turbulent flow.

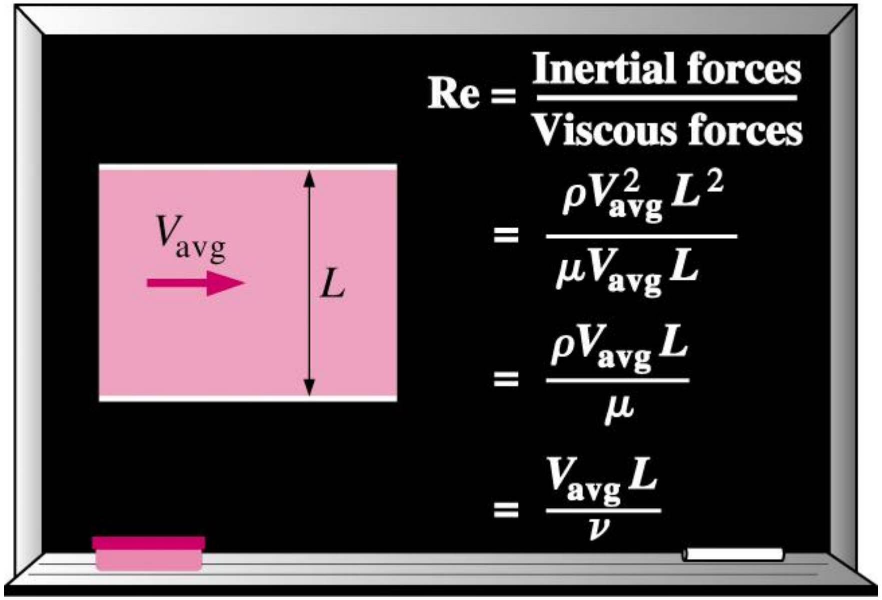


No analytical solutions exist! (It is too complicated, again because of the 3-D, unsteady, chaotic swirling eddies.)

Occurs at *high* Reynolds numbers.

Laminar and Turbulent Flows

Definition of Reynolds number



- Critical Reynolds number (Re_{cr}) for flow in a round pipe

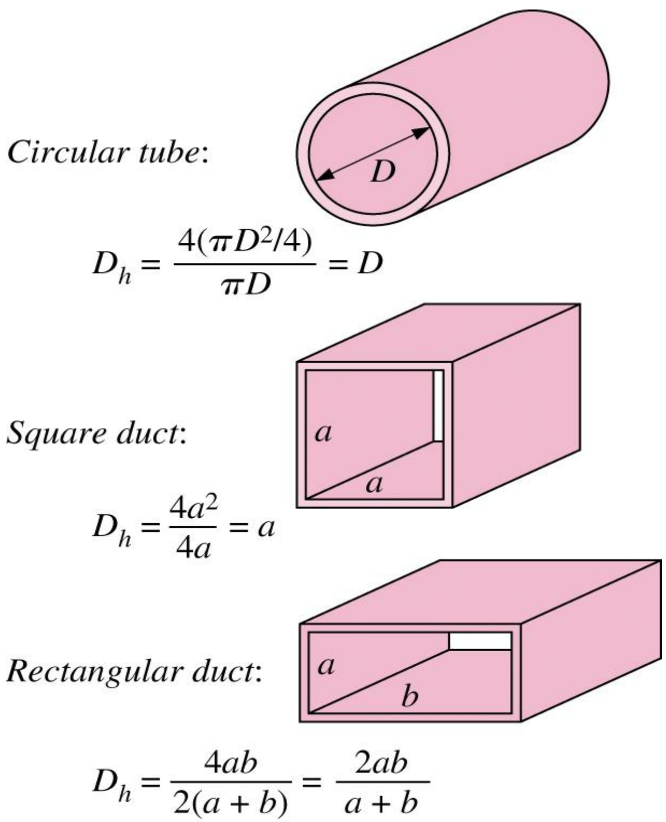
  Re < 2300 $\Rightarrow$ laminar

  2300 ≤ Re ≤ 4000 $\Rightarrow$ transitional

  Re > 4000 $\Rightarrow$ turbulent

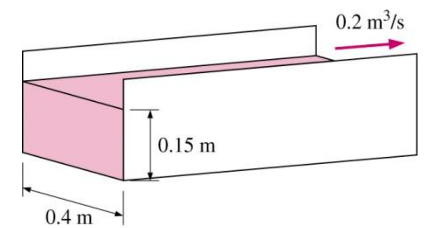
- Note that these values are approximate.
- For a given application, Recr depends upon
  - Pipe roughness
  - Vibrations
  - Upstream fluctuations, disturbances (valves, elbows, etc. that may disturb the flow)

Laminar and Turbulent Flows



- For <u>non-round</u> pipes, define the hydraulic diameter
  $D_h = 4A_c/P$
  A_c = cross-section area
  P = wetted perimeter

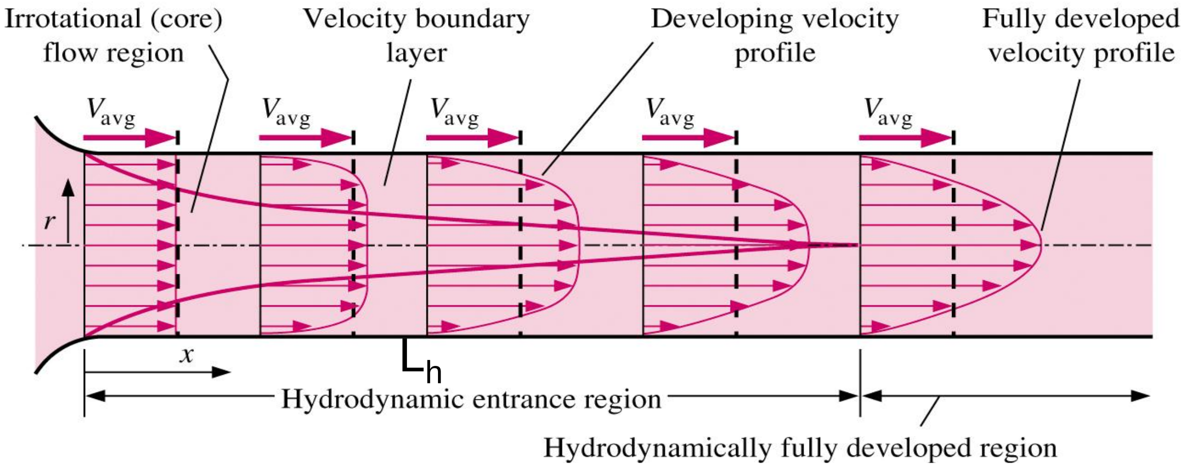
- Example: open channel



  $A_c = 0.15 * 0.4 = 0.06\text{m}^2$
  $P = 0.15 + 0.15 + 0.5 = 0.8\text{m}$
  Don't count free surface, since it does not contribute to friction along pipe walls!
  $D_h = 4A_c/P = 4*0.06/0.8 = 0.3\text{m}$
  <u>What does it mean?</u> This channel flow is equivalent to a round pipe of diameter 0.3m (approximately).

The Entrance Region

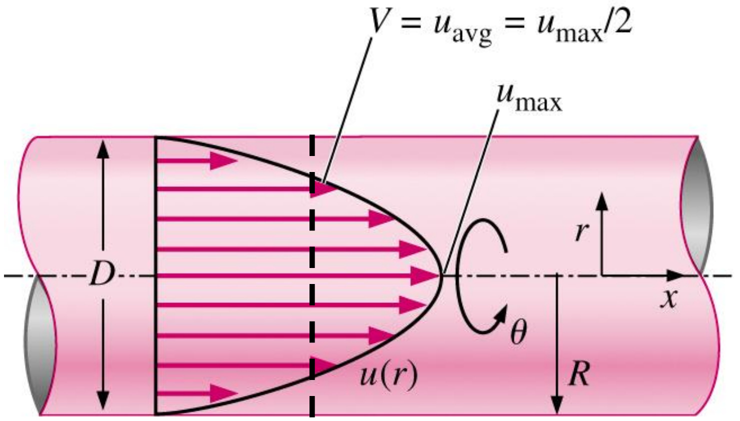
- Consider a round pipe of diameter D. The flow can be laminar or turbulent. In either case, the profile develops downstream over several diameters called the *entry length* L_h. L_h/D is a function of Re.



Fully Developed Pipe Flow

- Comparison of laminar and turbulent flow

There are some major differences between laminar and turbulent fully developed pipe f



<u>Laminar</u>

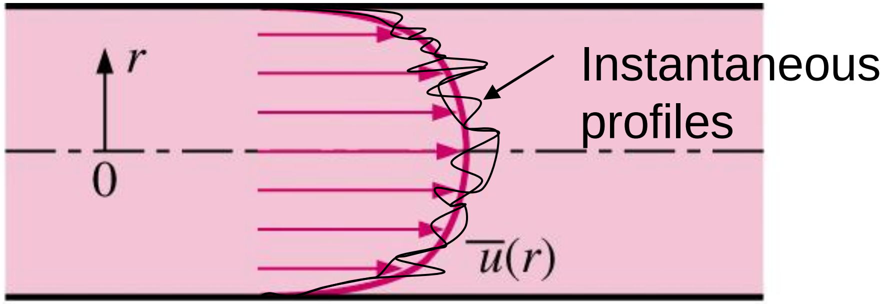
- Can solve exactly
- Flow is steady
- Velocity profile is parabolic
- Pipe roughness not important

It turns out that $V_{avg} = 1/2U_{max}$ and $u(r)= 2V_{avg}(1 - r^2/R^2)$

Fully Developed Pipe Flow

Turbulent

- *Canno*t solve exactly (too complex)
- Flow is unsteady (3D swirling eddies), but it is steady in the mean
- Mean velocity profile is fuller (shape more like a top-hat profile, with very sharp slope at the wall)
- Pipe roughness is very important



- V_{avg} 85% of U_{max} (depends on Re a bit)
- No analytical solution, but there are some good semi-empirical expressions that approximate the velocity profile shape.

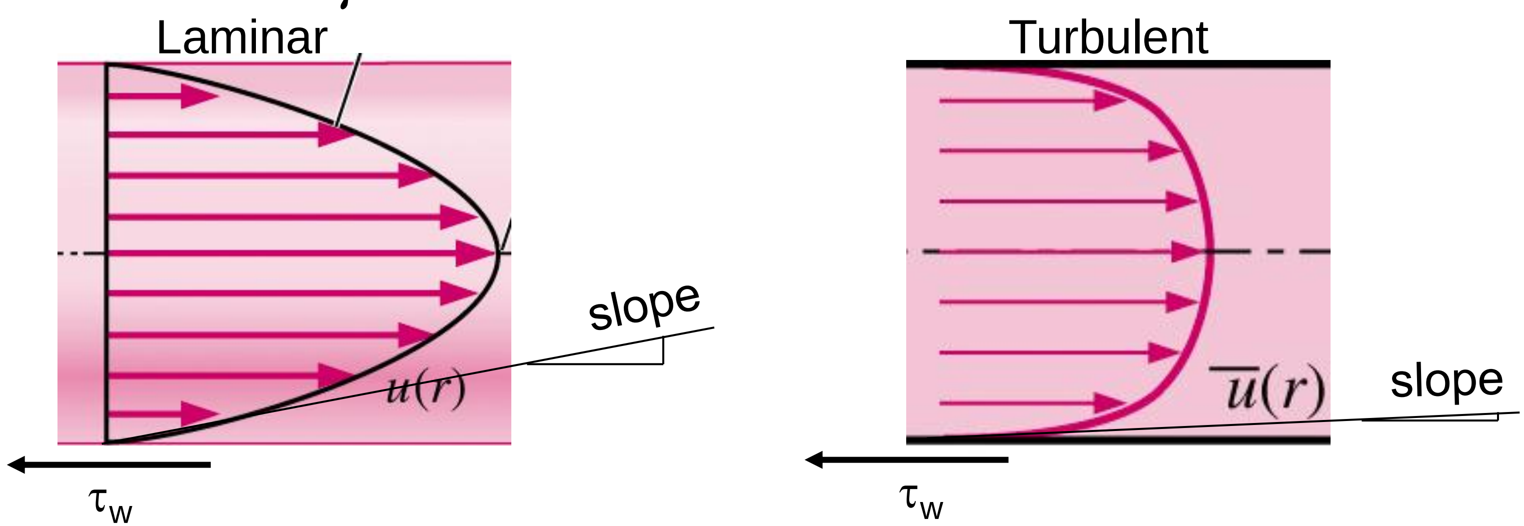
Fully Developed Pipe Flow Wall-shear stress

- Recall, for simple shear flows $u=u(y)$, we had

$$\tau = \mu du/dy$$

- In fully developed pipe flow, it turns out that

$$\tau = \mu du/dr$$

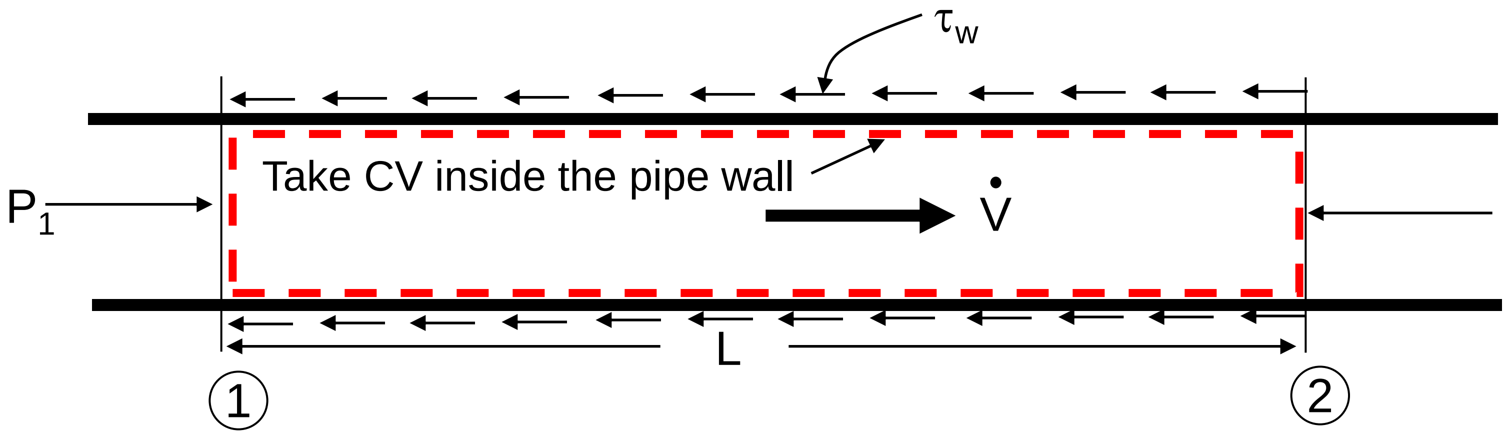


τ_w = shear stress at the wall, acting on the fluid

$$\boxed{\tau_{w,turb} > \tau_{w,lam}}$$

Fully Developed Pipe Flow Pressure drop

- There is a direct connection between the pressure drop in a pipe and the shear stress at the wall
- Consider a <u>horizontal pipe</u>, fully developed, and incompressible flow



- Let’s apply conservation of mass, momentum, and energy to this CV

Fully Developed Pipe Flow Pressure drop

- <u>Conservation of Mass</u>

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$\rho\dot{V}_1 = \rho\dot{V}_2 \rightarrow \dot{V} = const$$

$$V_1\frac{\pi D^2}{4} = V_2\frac{\pi D^2}{4} \rightarrow \boxed{V_1 = V_2}$$

- <u>Conservation of x-momentum</u>

$$\sum F_x = \cancel{\sum F_{x,grav}} + \sum F_{x,press} + \sum F_{x,visc} + \cancel{\sum F_{x,other}} = \sum_{out}\beta\dot{m}V - \sum_{in}\beta\dot{m}V$$

$$P_1\frac{\pi D^2}{4} - P_2\frac{\pi D^2}{4} - \tau_w \pi D L = \underbrace{\cancel{\beta_2\dot{m}V_2} - \cancel{\beta_1\dot{m}V_1}}$$

Terms cancel since $\beta_1 = \beta_2$ and $V_1 = V_2$

Fully Developed Pipe Flow Pressure drop

- Thus, x-momentum reduces to

$$(P_1 - P_2)\frac{\pi D^2}{4} = \tau_w \pi D L$$

or

$$\boxed{P_1 - P_2 = 4\tau_w \frac{L}{D}}$$

- Energy equation (in head form)

$$\frac{P_1}{\rho g} + \cancel{\alpha_1 \frac{V_1^2}{2g}} + \underbrace{\cancel{z_1} + \cancel{h_{pump,u}} = \frac{P_2}{\rho g} + \cancel{\alpha_2 \frac{V_2^2}{2g}} + \cancel{z_2}}_{\text{cancel (horizontal pipe)}} + \cancel{h_{turbine,e}} + h_L$$

Velocity terms cancel again because $V_1 = V_2$, and $\alpha_1 = \alpha_2$ (shape not changing)

$$\boxed{P_1 - P_2 = \rho g h_L}$$

h_L = irreversible head loss & it is felt as a pressure drop in the pipe

Fully Developed Pipe Flow Friction Factor

- From momentum CV analysis $$\boxed{P_1 - P_2 = 4\tau_w \frac{L}{D}}$$
- From energy CV analysis $$\boxed{P_1 - P_2 = \rho g h_L}$$
- Equating the two gives
$$4\tau_w \frac{L}{D} = \rho g h_L \qquad \boxed{h_L = \frac{4\tau_w}{\rho g} \frac{L}{D}}$$
- To predict head loss, we need to be able to calculate τ_w. How?
  - Laminar flow: solve exactly
  - Turbulent flow: rely on empirical data (experiments)
  - In either case, we can benefit from dimensional analysis!

Fully Developed Pipe Flow Friction Factor

- τ_w = func(ρ, V, μ, D, ε)

ε = average roughness of the inside wall of the pipe

- Π-analysis gives

$$\Pi_1 = f$$

$$\Pi_2 = Re$$

$$\Pi_3 = \frac{\epsilon}{D}$$

$$\boxed{f = \frac{8\tau_w}{\rho V^2}}$$

$$\boxed{Re = \frac{\rho V D}{\mu}}$$

$$\boxed{\epsilon/D = \text{roughness factor}}$$

$$\Pi_1 = func(\Pi_2, \Pi_3)$$

$$\boxed{f = func(Re, \epsilon/D)}$$

Fully Developed Pipe Flow Friction Factor

- Now go back to equation for h_L and substitute f for τ_w

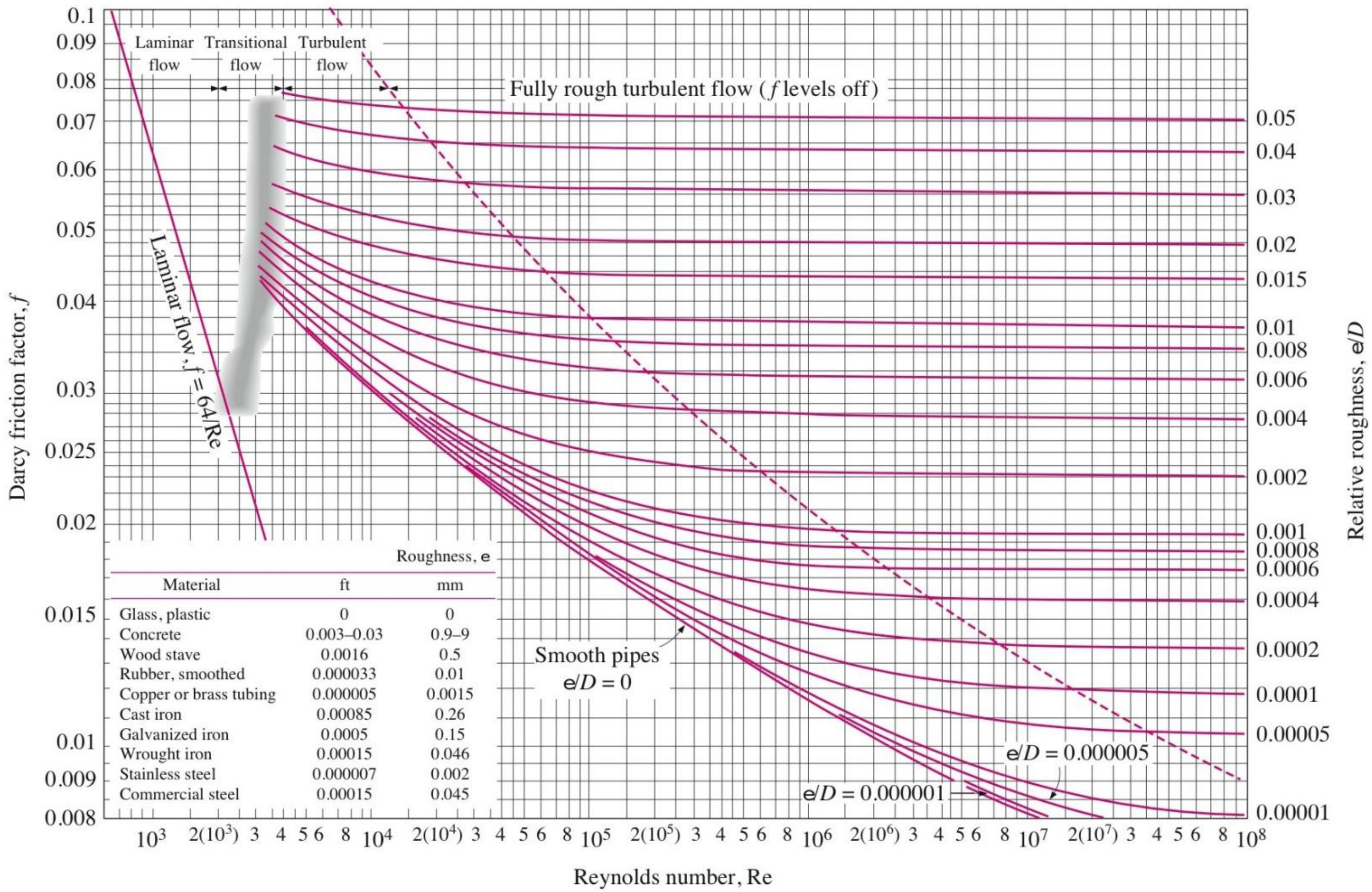
$$h_L = \frac{4\tau_w}{\rho g}\frac{L}{D} \qquad f = \frac{8\tau_w}{\rho V^2} \rightarrow \tau_w = f\rho V^2/8$$

$$\boxed{h_L = f\frac{L}{D}\frac{V^2}{2g}}$$

- Our problem is now reduced to solving for Darcy friction factor f
  - Recall $f = func(Re, \epsilon/D)$
  - Therefore
    - Laminar flow: f = 64/Re (exact)
    - Turbulent flow: Use charts or empirical equations (Moody Chart, a famous plot of f vs. Re and ε/D)

But for laminar flow, roughness does not affect the flow unless it is huge

The Moody Chart



	Roughness, e	
Material	ft	mm
Glass, plastic	0	0
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

Fully Developed Pipe Flow Friction Factor

- Moody chart was developed for circular pipes, but can be used for non-circular pipes using hydraulic diameter
- Colebrook equation is a curve-fit of the data which is convenient for computations

$$\frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{\epsilon / D}{3.7} + \frac{2.51}{Re\sqrt{f}} \right)$$

Implicit equation for f which can be solved using the root-finding algorithm

- Both Moody chart and Colebrook equation are accurate to ±15% due to roughness size, experimental error, curve fitting of data, etc.

Types of Fluid Flow Problems

- In design and analysis of piping systems, 3 problem types are encountered

1. Determine Δp (or h_L) given L, D, V (or flow rate)

   Can be solved directly using Moody chart and Colebrook equation

2. Determine V, given L, D, Δp
3. Determine D, given L, Δp, V (or flow rate)

- Types 2 and 3 are common engineering design problems, i.e., selection of pipe diameters to minimize construction and pumping costs
- However, iterative approach required since both *V* and *D* are in the Reynolds number.

Types of Fluid Flow Problems

- Explicit relations have been developed which eliminate iteration. They are useful for quick, direct calculation, but introduce an additional 2% error

$$h_L = 1.07\frac{\dot{\mathcal{V}}^2 L}{gD^5}\left\{\ln\left[\frac{\epsilon}{3.7D} + 4.62\left(\frac{\nu D}{\dot{\mathcal{V}}}\right)^{0.9}\right]\right\}^{-2} \qquad \begin{array}{c} 10^{-6} < \epsilon/D < 10^{-2} \\ 3000 < Re < 3\times 10^{8} \end{array}$$

$$\dot{\mathcal{V}} = -0.965\left(\frac{gD^5 h_L}{L}\right)^{0.5}\ln\left[\frac{\epsilon}{3.7D} + \left(\frac{3.17\nu^2 L}{gD^3 h_L}\right)^{0.5}\right] \qquad Re > 2000$$

$$D = 0.66\left[\epsilon^{1.25}\left(\frac{L\dot{\mathcal{V}}^2}{gh_L}\right)^{4.75} + \nu\dot{\mathcal{V}}^{9.4}\left(\frac{L}{gh_L}\right)^{5.2}\right]^{0.04} \qquad \begin{array}{c} 10^{-6} < \epsilon/D < 10^{-2} \\ 5000 < Re < 3\times 10^{8} \end{array}$$

Minor Losses

- Piping systems include fittings, valves, bends, elbows, tees, inlets, exits, enlargements, and contractions.
- These components interrupt the smooth flow of fluid and cause additional losses because of flow separation and mixing
- We introduce a relation for the minor losses associated with these components

$$\boxed{h_L = K_L \frac{V^2}{2g}}$$

- K_L is the loss coefficient.
- Is different for each component.
- Is assumed to be independent of Re.
- Typically provided by manufacturer or generic table.

Minor Losses

- Total head loss in a system is comprised of major losses (in the pipe sections) and the minor losses (in the components)

$$h_L = h_{L,major} + h_{L,minor}$$

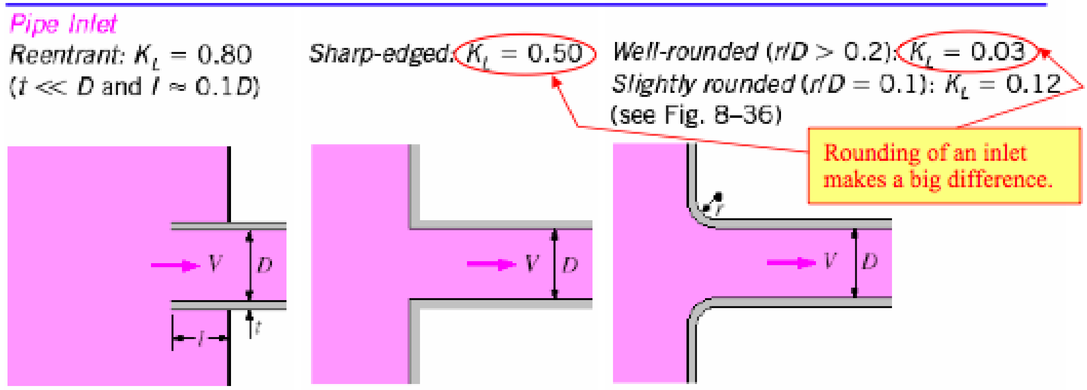
$$h_L = \underbrace{\sum_i f_i \frac{L_i}{D_i}\frac{V_i^2}{2g}}_{\text{i pipe sections}} + \underbrace{\sum_j K_{L,j}\frac{V_j^2}{2g}}_{\text{j components}}$$

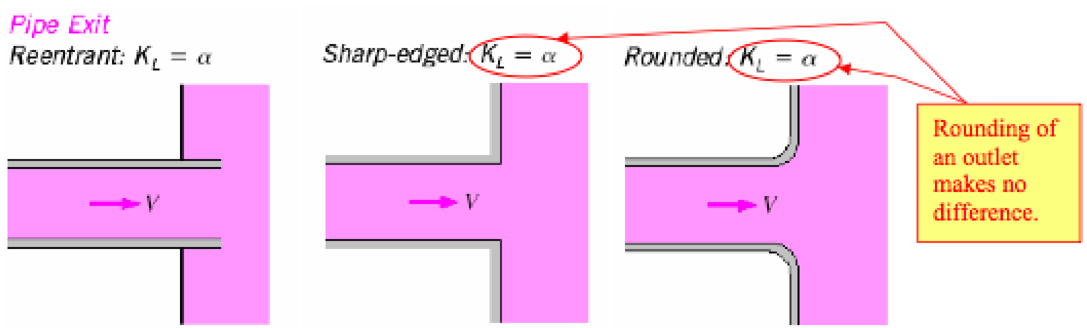
- If the piping system has constant diameter

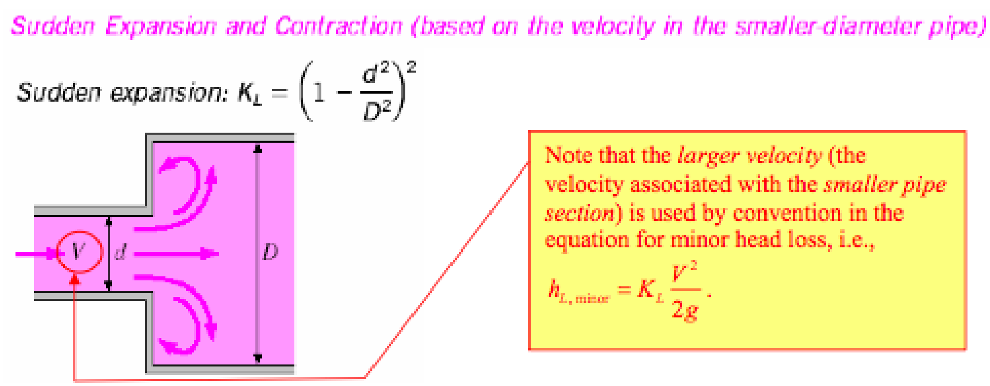
$$h_L = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g}$$

Minor Losses

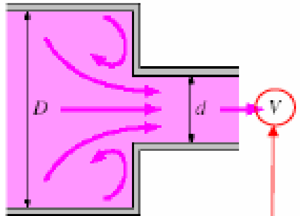
Here are some sample loss coefficients for various minor loss components. More values are listed in Table 8-4, page 350 of the Çengel-Cimbala textbook:

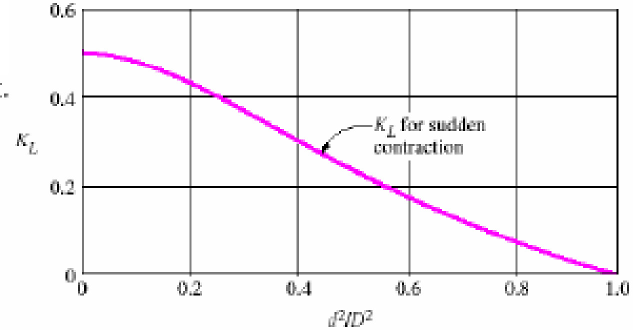






Sudden contraction: See chart.





Note: These are backwards. The K_L values listed for Expansion should be those for Contraction, and vice-versa.

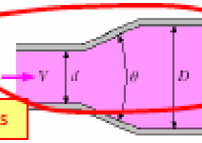
Note again that the *larger velocity* (the velocity associated with the *smaller pipe section*) is used by convention in the equation for minor head loss, i.e., $h_{L,\,minor} = K_L \dfrac{V^2}{2g}$.

Gradual Expansion and Contraction (based on the velocity in the smaller-diameter pipe)

Expansion:

$K_L = 0.02$ for $\theta = 20°$
$K_L = 0.04$ for $\theta = 45°$
$K_L = 0.07$ for $\theta = 60°$

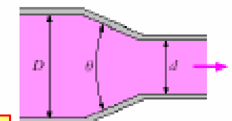
These are for contractions



Contraction (for $\theta = 20°$):

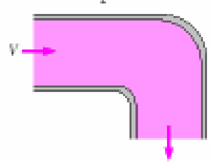
$K_L = 0.30$ for $d/D = 0.2$
$K_L = 0.25$ for $d/D = 0.4$
$K_L = 0.15$ for $d/D = 0.6$
$K_L = 0.10$ for $d/D = 0.8$

These are for expansions

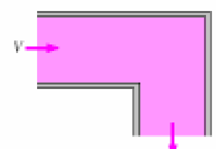


Bends and Branches

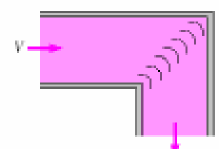
90° smooth bend:
Flanged: $K_L = 0.3$
Threaded: $K_L = 0.9$



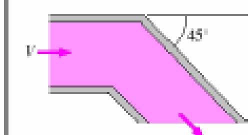
90° miter bend
(without vanes): $K_L = 1.1$



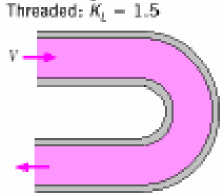
90° miter bend
(with vanes): $K_L = 0.2$



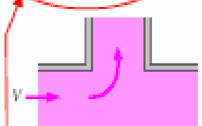
45° threaded elbow:
$K_L = 0.4$



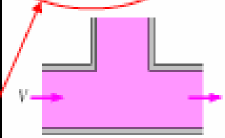
180° return bend:
Flanged: $K_L = 0.2$
Threaded: $K_L = 1.5$



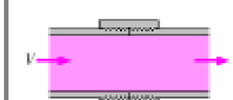
Tee (branch flow):
Flanged: $K_L = 1.0$
Threaded: $K_L = 2.0$



Tee (line flow):
Flanged: $K_L = 0.2$
Threaded: $K_L = 0.9$



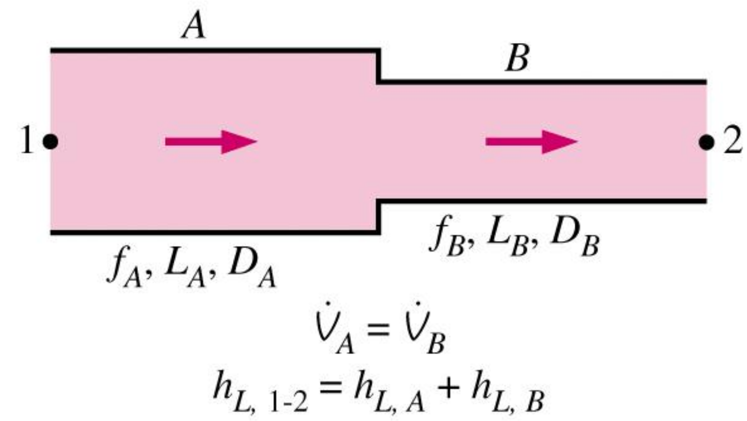
Threaded union:
$K_L = 0.08$

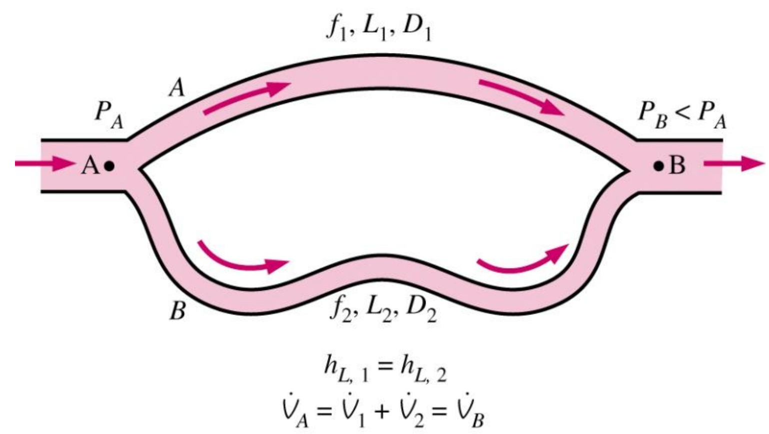


For tees, there are two values of K_L, one for *branch flow* and one for *line flow*.

Piping Networks and Pump Selection

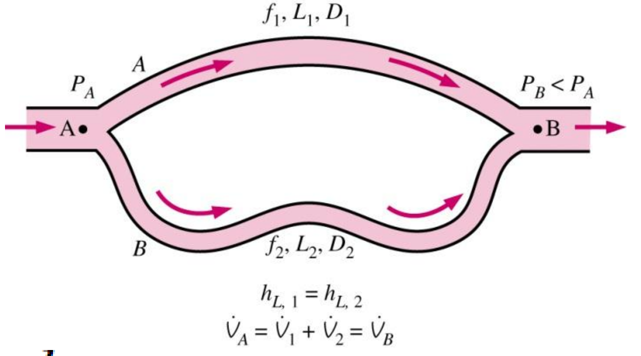
- Two general types of networks
  - Pipes in series
    - Volume flow rate is constant
    - Head loss is the summation of parts
  - Pipes in parallel
    - Volume flow rate is the sum of the components
    - Pressure loss across all branches is the same





Piping Networks and Pump Selection

- For parallel pipes, perform CV analysis between points A and B



$$V_A = V_B$$

$$\frac{P_A}{\rho g} + \cancel{\alpha_1 \frac{V_A^2}{2g}} + \cancel{z_A} = \frac{P_B}{\rho g} + \cancel{\alpha_2 \frac{V_B^2}{2g}} + \cancel{z_B} + h_L$$

$$h_L = \frac{\Delta P}{\rho g}$$

- Since Δp is the same for all branches, head loss in all branches is the same

$$h_{L,1} = h_{L,2} \Longrightarrow f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} = f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g}$$

Piping Networks and Pump Selection

- Head loss relationship between branches allows the following ratios to be developed

$$\frac{V_1}{V_2} = \left(\frac{f_2}{f_1}\frac{L_2}{L_1}\frac{D_1}{D_2}\right)^{\frac{1}{2}} \qquad \frac{\dot{\mathcal{V}}_1}{\dot{\mathcal{V}}_2} = \frac{D_1^2}{D_2^2}\left(\frac{f_2}{f_1}\frac{L_2}{L_1}\frac{D_1}{D_2}\right)^{\frac{1}{2}}$$

- Real pipe systems result in a system of non-linear equations.
- Note: the analogy with electrical circuits should be obvious
  - Flow flow rate (VA) : current (I)
  - Pressure gradient (Δp) : electrical potential (V)
  - Head loss (h_L): resistance (R), however h_L is very nonlinear

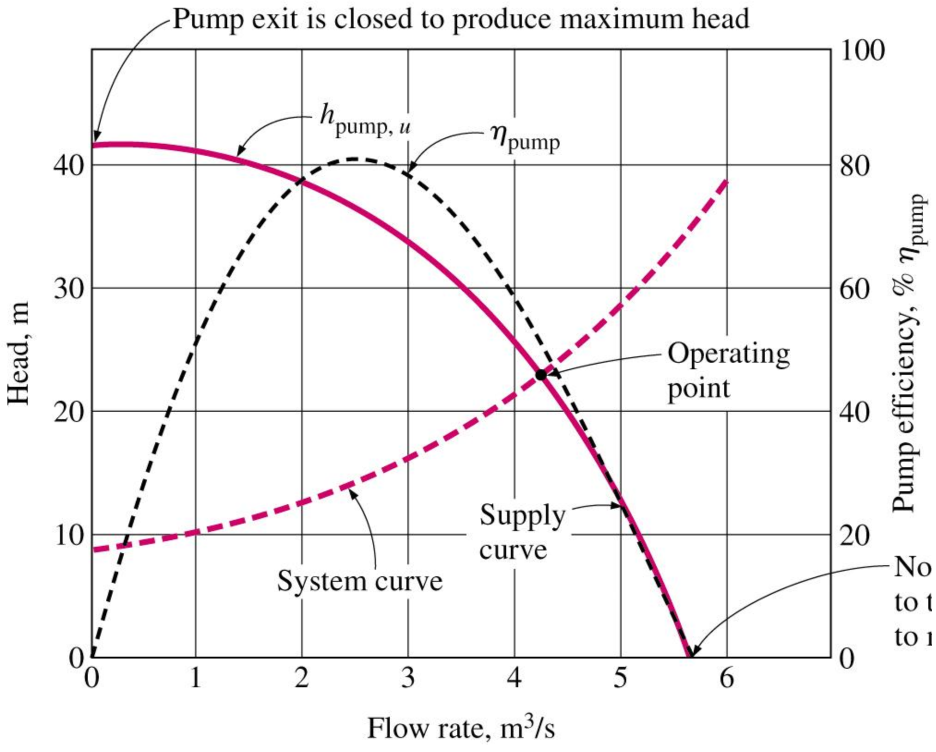
Piping Networks and Pump Selection

- When a piping system involves pumps and/or turbines, pump and turbine head must be included in the energy equation

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + \boxed{h_{pump,u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + \boxed{h_{turbine,e}} + h_L$$

- The useful head of the pump ($h_{pump,u}$) or the head extracted by the turbine ($h_{turbine,e}$), are functions of volume flow rate, i.e., they are not constants.
- Operating point of system is where the system is in balance, e.g., where pump head is equal to the head losses.

Pump and systems curves



- Supply curve for $h_{pump,u}$: determine experimentally by manufacturer. It is easy to build in functional relationship for $h_{pump,u}$.
- System curve determined from analysis of fluid dynamics equations
- Operating point is the intersection of supply and demand curves
- If peak efficiency is far from operating point, pump is wrong for that application.